STRUCTURE OF HIGH-PRESSURE LOW-DENSITY JETS BEYOND A SUPERSONIC NOZZLE

N. I. Kislyakov, A. K. Rebrov,
and R. G. Sharafutdinov

UDC 533.697+533.6.011.8

The results are presented on an experimental study of the structure of high-pressure low-density jets beyond a supersonic nozzle when the gas escapes into a quiescent medium.

Many reports have been devoted to jets of a viscous gas beyond supersonic nozzles. Until recently, however, in the majority of cases turbulent jets with a small nonratedness (the ratio of pressures at the nozzle cut and in the surrounding gas $n = p_a/p_\infty < 100$) have been studied. An idea of the level and the basic results of the studies of jets with a large nonratedness is given by [1-5].

The qualitative properties of the variation in the structure of the jet with an increase in the rarefaction (thickening of the mixing zones and the shock-wave zones, strengthening of the dissipative processes in the core of the jet, rearrangement of the flow) are well known at present from studies of jets beyond a sonic nozzle (see [6] and the references cited there) in the transitional region of flows up to the mode of scattering [7]. The picture of the flow in low-density jets beyond a supersonic nozzle is completed by [4, 5].

In the present report on the basis of measurments of the density in N_2 and CO_2 jets quantitative data are obtained on the flow structure in modes from laminar continuous flow to strongly rarefied flow, the possibility of generalization of the experimental data is demonstrated, and the structure of the jet is studied in the region of the Mach disk and the X-shaped configuration.

1. Considerations on the Similarity of Jets

The general questions of the similarity of jets are considered in [1, 8-10]. An earlier result in the analysis of the similarities of jets is the demonstration of the self-similarity of the geometrical configuration of a jet with respect to the nonratedness.

For concrete conditions of the expansion of a gas it is sometimes necessary to know the nature of the approximation to self-similarity with respect to the nonratedness and the effect of the viscosity on the geometry of the jet and the distribution of the parameters. For an answer to the first question the results of calculations of jets of a nonviscous gas are analyzed below. The search for the solution of the second question comprised the main content of the experimental studies described in the subsequent sections.

We will assume that the calculated structure of a jet of nonviscous gas is the asymptotically limiting structure for the laminar flow upon an increase in the chracteristic Reynolds number (without a transition to turbulent flow) and can be used as the reference structure in the analysis of the effect of viscosity. As is known, when $n \gg 1$ the distance along the jet axis from the nozzle cut to the closing shock wave is proportional to the value of the complex $M_a\sqrt{\gamma n}$, where γ is the adiabatic index and M_a is the Mach number at the nozzle cut. Let us examine the variation in the transverse dimensions of a jet (the diameters of the suspended wave and of the jet boundaries) based on the data of the numerical calculation of [11]. Strict self-similarity is not observed at distances of $x/d_a\sqrt{n} > 0.5$ along the axis from the nozzle cut. In the dimensionless coordinates $x/d_a\sqrt{n}$, $y/d_a\sqrt{n}$ (y is the distance from the axis of the jet) the contours of the suspended shock waves are markedly different for $x/d_a\sqrt{n} > 0.5$. One can speak of the approximate proportionality of the transverse dimensions to the value n^α, where $\alpha < 0.5$; $\alpha \to 0.5$ as $n \to \infty$. The nature of this

---

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 42-52, March-April, 1975. Original article submitted February 28, 1974.

TABLE 1

Mode No.	$p_a \cdot 10^{-6}$	p_∞	T_0	n	Re_L	$(N/Z)^*$
			CO_2			
1a	3,19	13,3	680	714	213	300
b	3,21	8,8	680	1090	171	300
c	3,21	4,0	695	2400	116	300
d	3,21	2,13	690	4500	86	300
e	3,21	1,56	690	6540	70,5	300
f	3,21	0,91	690	10000	58,3	300
g	3,27	0,13	710	73000	8,4	300
h	0,47	12,5	550	101	109	30
i	1,03	13,3	590	232	148	120
j	1,95	13,3	700	437	166	100
2a	6	19,9	715	886	380	1550
b	5,76	19,6	610	860	361	2500
c	5,4	18,4	720	870	348	1000
d	4,9	18,2	720	835	320	800
e	4,45	15,1	720	860	288	600
f	4,0	13,7	710	860	266	550
g	3,4	11,7	710	860	228	500
h	1,45	5	605	856	107	140
i	0,720	2,49	605	860	53	40
j	0,36	1,25	580	860	27,6	12
3a	3,02	8,9	600	1000	217	900
b	0,955	2,61	293	1090	224	28 000
			N_2			
4a	1,78	5,98	680	580	92	100
b	1,25	9,15	700	265	94	30
c	0,82	13,3	715	120	93	7
d	1,78	3,06	680	1140	66	100
e	1,25	3,93	700	620	60	30
f	1,25	4,72	730	515	66	28
g	0,82	6,91	750	231	66	5
h	1,78	1,66	700	2100	37	80
i	1,25	2,39	700	1000	37	30
j	0,82	3,33	730	480	35,5	7
5a	1,2	9,3	293	212	265	35 000
b	0,56	3,93	550	229	52	30
c	1,2	9,2	750	214	85	18
d	1,97	15,3	950	210	104	7

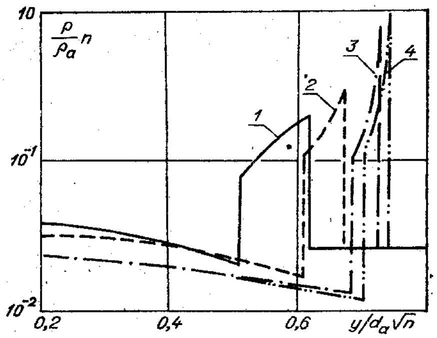

Fig. 1

asymptotic behavior is illustrated in Fig. 1 with an example from the atlas of [11], where the distribution of the relative density $(\rho/\rho_a)\cdot n$ over the normalized transverse coordinate is given for the cross section with $\bar{x}^1 = x/0.7d_a M_a\sqrt{\gamma n} = 0.5$ for $M_a = 4$, $\gamma = 1.3$, and $\Theta = 10°$: 1) $n = 10$; 2) 10^2; 3) 10^4; 4) 10^6 (Θ is the half-aperture angle of the nozzle). The maximum radius of the suspended shock wave can be expressed by the approximate dependence

$$r/d_a\sqrt{n} = 0.725 - 0.325/n^{0.25}, \tag{1.1}$$

from which it follows, for example, that for $n \approx 250$ such jets are self-similar with an accuracy of 10% of the variation in the maximum transverse dimensions.

A similar tendency is traced from the results of the calculation of [12] for $M_a = 4$, $\gamma = 1.4$, $\Theta = 10°$, and $n = 10\text{–}10^7$, with the only difference that in Eq. (1.1) the maximum value of $r/d_a\sqrt{n}$ as $n \to \infty$ is 0.68.

The effect of M_a and γ on the maximum cross-sectional radius of the shock wave can be taken into account by using a dependence which follows from the calculations of É. A. Ashratov:

$$r_a/d_a\sqrt{n} \sim \left(1 - 0.38/M_a^{1.5}\right)/\gamma. \tag{1.2}$$

Unfortunately, there are not enough calculated data to establish a general dependence of the type of (1.1) for different nozzles and working substances with an indication of the permissible limits of its use with respect to M_a, n, Θ, and γ. Therefore the dependence (1.1) together with the correction (1.2) for conditions differing from those under consideration can be used only in an estimate of the approximation to self-similarity with variation in the nonratedness.

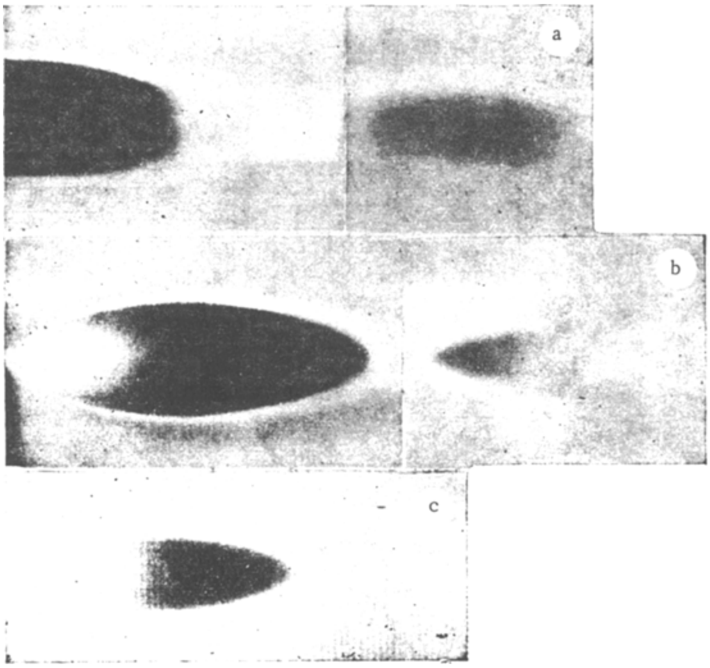

Fig. 2

The effect of the viscosity leads to a shift in the suspended shock-wave front and a change in the conditions of self-similarity of the geometry of the jet with respect to the nonratedness. For a description of viscous effects in the mixing zone in the initial section of a jet one can use the characteristic local Reynolds number in the form [2]

$$\mathrm{Re} = \frac{w L_M \rho_{av}}{\mu_{av}},$$

where L_M is the distance from the nozzle cut to the Mach disk; w is the maximum velocity with respect to the stagnation parameters; μ_{av} and ρ_{av} are the average values of the dynamic viscosity and density in the zone of mixing of the jet with the surrounding gas. For a constant M_a under integral-adiabatic conditions ($T_0 = T_\infty$) this criterion is directly proportional to the complex $\mathrm{Re}_L = \mathrm{Re}_* / [p_c/p_\infty]^{-1/2}$ where Re_* is the Reynolds number at the critical cross section. In [6] it is shown that the complex Re_L characterizes the effect of the viscosity in all zones of the jet. The proportionality factor between Re and Re_L is some function of M_a and T_0/T_∞. Later we will use the complex Re_L as a determining complex.

2. Conditions of Performance of Experiments

The studies of the present work were performed on a low-density gasdynamic installation with combined evacuation by boosters and cryogenic vacuum pumps. The set-up of the installation is briefly described in [13]. Cryogenic pumps operating on liquid nitrogen were used to expand the range in nonratedness and in Reynolds numbers. With this the capacity of the installation (for CO_2 as the working gas) increased by an order of magnitude and reached 7 g/sec at a pressure of 1 N/m^2 in the vacuum chamber.

The study of low-density jets in the present work is based on the measurement of the density and visualization of the flow pattern using an electron beam. The spectral sections of 3900 ± 25 Å for nitrogen and 2876 ± 20 Å for CO_2 were chosen for the density measurements. There are data for the selection of these sections in [14, 15]. The error in the density determination did not exceed 8% for direct measurement and was no more than 14% with photometry.

All the experiments were performed on a conical nozzle with a diameter of 0.51 ± 0.01 mm at the critical cross section and 2.46 mm at the cut and with a half-aperture angle of 10°. The Mach number at the nozzle cut was determined with a Pitot tube from the parameters of equilibrium expansion. The experimental conditions are presented in Table 1.

The modes of flow studied correspond to: 1a-g) variation of nonratedness with a constant CO_2 flow rate; 2a-j) variation of CO_2 flow rate with a constant nonratedness; 4a-j) a series of experiments on nitro-

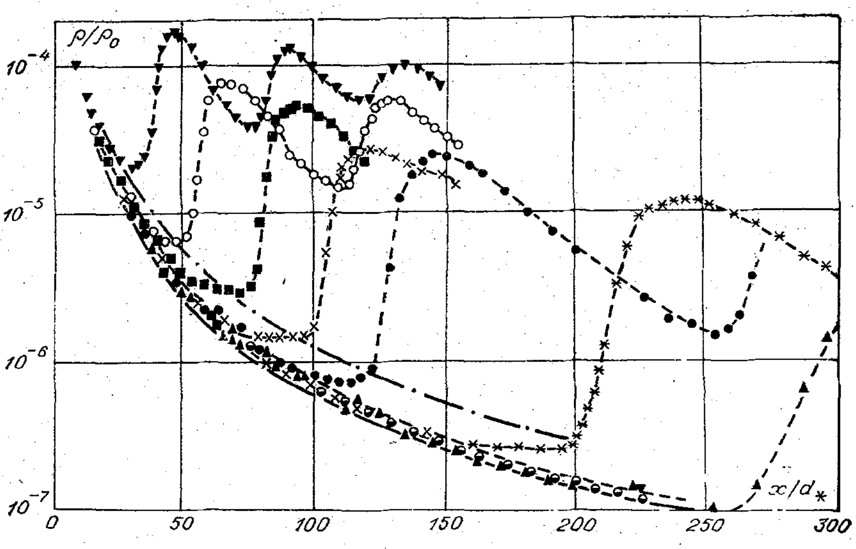

Fig. 3

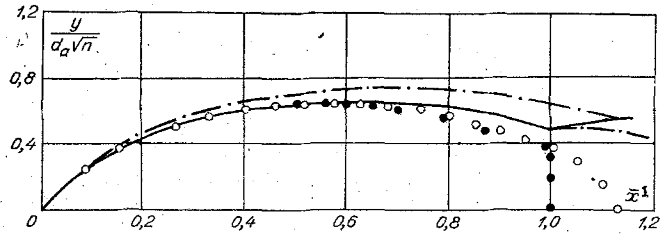

Fig. 4

gen with constant values of the complex Re_L. The effect of condensation was detected in modes 3a and b and 5a and the effect of the temperature factor on the structure of the nitrogen jet was detected in modes 5a-d. In Table 1 (N/Z)* is the ratio of the number of molecules in a cluster to its charge.

3. General Structure of Jet. Transition to an X-Shaped Configuration

Let us examine the change in the structure of a jet upon a reduction in the density level on the example of the expansion of CO_2 from a nozzle with $M_a = 4.35$. A plane axial cross section of the jet for a length of two cycles is shown in Fig. 2 for the conditions $p_a/p_\infty = 860$ and $T_0/T_\infty = 2$-2.4 (modes 2a-j). Since the temperature factor varies insignificantly, the effect of the rarefaction will be characterized by the number Re_L.

In a relatively dense gas when $Re_L = 380$ (see Fig. 2a) the initial section of the jet is closed by the Mach disk, after which the second cycle is formed. In the range of $Re_L = 380$-348 a transition occurs to a new configuration having the form of a "regular reflection." We will call it the X-shaped configuration, as opposed to the Mach configuration. In Fig. 2b ($Re_L = 320$) a structure containing shock waves is observed for a length of two "barrels." With a decrease in Re_L the dimensions of the core are reduced, the structure becomes blurred, and by the time $Re_L = 27.6$ (see Fig. 2c) shock waves do not appear in the picture in the second cycle and only diffuse density clusters are seen; the first cycle retains its structure.

The density distribution along the axis of the jet for modes 1a-g (a is the minimum and g is the maximum nonratedness) is shown in Fig. 3. The number Re_* is kept constant and therefore the effect of the nonratedness is traced. One of the properties of a jet beyond a supersonic nozzle is displayed here – an increase in density in the region of joining of the shock waves, considerably exceeding the density increase in a shock wave, as well as the early and considerable departure of the density dependence from isentropy, caused by the effect of the lateral shock-wave fronts.

The contours of the shock waves in the modes with $Re_L = 380$ (in the presence of a Mach disk) and $Re_L = 348$ (with an X-shaped configuration) are shown in Fig. 4 in generalized coordinates. The position of the shock wave was determined from the points with the maximum density gradient. The contours of the surfaces of a jet of nonviscous gas (closest in parameters to the jet studied) with $N = 10^5$, $M_{av} = 4$, $\Theta = 10°$, and $\gamma = 1.3$ from a series of contours calculated in [11] are also presented here. Under actual conditions the suspended shock wave is forced back toward the axis. The position of the Mach disk coincides with that for a nonviscous gas. The appearance of an X-shaped configuration does not introduce changes in the position of the suspended shock wave and only the distance to the shock waves along the axis of the jet is increased. The theory of a nonviscous fluid [16, 17] does not admit of the regular reflection of a shock wave in an axisymmetric stream. With a change in the conditions in the direction of a transition to regular reflection in a nonviscous fluid the diameter of the Mach disk decreases without limit, asymptotically approaching zero. The existence of a Mach configuration is connected with the presence of an annular contact discontinuity behind the Mach disk. Under actual conditions it may be removed owing to the viscous transfer of momentum. Thus, a configuration resembling regular reflection develops in a low-density gas stream. The formation of such a configuration has been noted in [5], for example, where the conditions of conversion to it were studied on the basis of the operating parameters. We should also note that in [5] this type of reflection was assumed a priori to be regular.

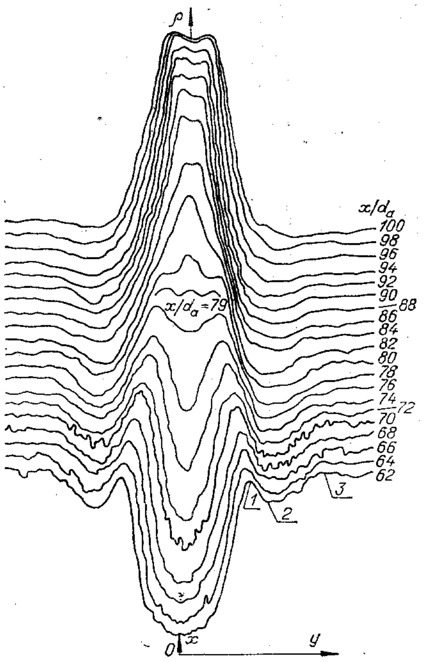

Fig. 5

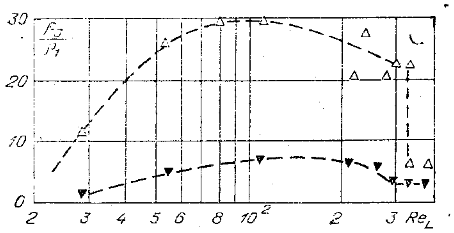

Fig. 6

In our experiments the visualization using an electron beam and the photography of a plane cross section of the jet with subsequent photometry made it possible to obtain the structure of the region of reflection of the shock wave in the vicinity of the jet axis. The pattern of the density field in mode 1j is shown in Fig. 5 as an example. The density distribution is given in cross sections equidistant from one another (every 2 calibers).

The first cross section passes through the core of the jet at $x/d_a = 62$; here the core is disturbed by the blurred suspended shock waves; the region 0−1 corresponds to an increase in density in the suspended shock wave; the region 1−2 corresponds to the change in density in the compressed layer; the boundary of the disturbances from the jet in the flooded space is very arbitrarily denoted by the number 3. The presence of a minimum in the vicinity of the point 2 is caused by the decrease in density as a consequence of the heating of the mixing zone upon the retardation in it of gas expanding from the source with a stagnation temperature of 700°K. The joining of the shock waves leads to an increase in the density in the suspended shock wave and in the compression layer ($x/d_a = 62$-78). In the cross section with $x/d_a = 79$ at the axis of the jet in about the region of intersection of the trailing fronts of the suspended shock waves there appears a zone of increases in density in the form of a third hump in the transverse distribution. This indicates the generation of reflected shock waves in the vicinity of the jet axis. In the successive cross sections along the axis of the jet there first appears a plateau and then a trough in the density distribution. Beyond the reflected shock wave the density continues to increase ($x/d_a = 86$-100) and the characteristic form of a curve with two maxima is produced. This irregularity is preserved downstream. In accordance with Fig. 3, for the mode of flow under consideration the minimum value of the dimensionless density in the region of the X-shaped configuration is $\rho_1/\rho_0 = 3 \cdot 10^{-6}$ and the maximum value is $\rho_3/\rho_0 = 5.3 \cdot 10^{-5}$; in the surrounding medium $\rho_\infty/\rho_0 = 1.6 \cdot 10^{-5}$. In the region of joining of the shock waves the Knudsen number, determined from the characteristic size − the distance between the density peaks, has the order of 0.01. Therefore the density irregularities are displayed rather clearly. In a jet with a Mach disk at the axis a density minimum should be observed everywhere in the vicinity of the Mach disk. The appearance of a maximum indicates the reflection of the shock waves, in contrast to the Mach case.

Let us dwell on a property mentioned earlier − the strong increase in density in the region of the X-shaped configuration. Calculations of the flow of a nonviscous gas show the existence of an increase in density on the streamline passing through the system of two oblique compression shock waves in the vicinity

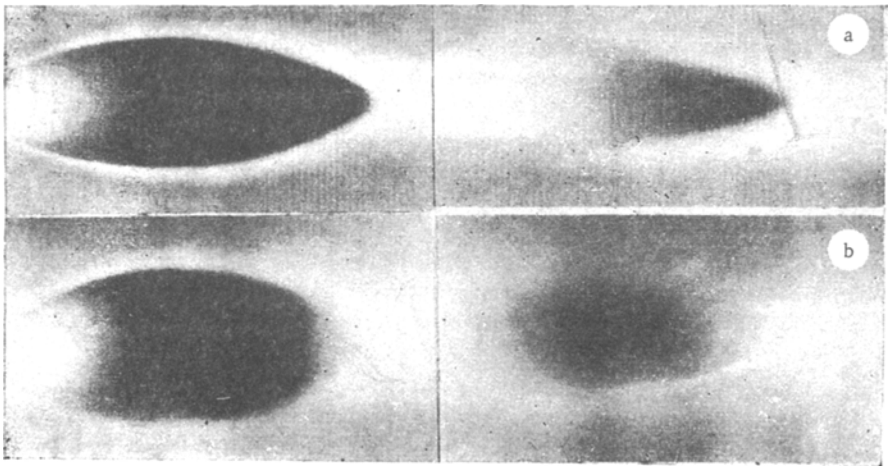

Fig. 7

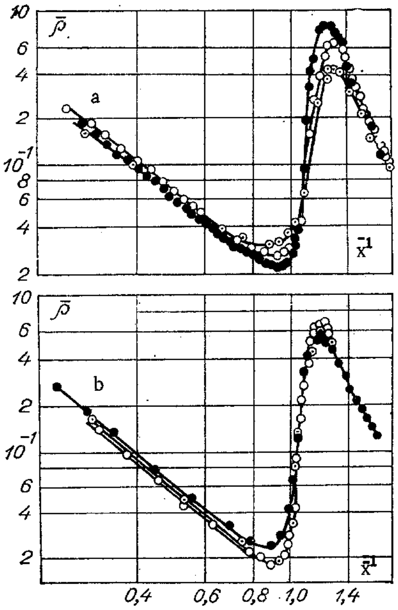

Fig. 8

of the Mach configuration; for example, for a jet beyond a nozzle with $M_a = 4$ with $n = 500$ and $\gamma = 1.3$ (based on the geometry given in the atlas of [11]) this proves to be a 22-fold increase in density. Experiments confirm the high order of the density increase.

The data for CO_2 with different Re_L in modes 2a-j are presented in Fig. 6. The jump in ρ_3/ρ_1 for $Re_L > 300$ is explained by the transition from a Mach to an X-shaped configuration. In modes with $Re_L > 200$ the density level in the region of the X-shaped configuration is very high and the correctness of its measurement may be placed under doubt; the electron beam measurements give understated values. Therefore it makes sense to talk about the behavior of the curve in the region of $Re_L < 200$. The decrease in ρ_3/ρ_1 with a decrease in Re_L is explained by the general blurring of the characteristic zones of the jet with an increase in the rarefaction. The dependence of ρ_3/ρ_1 shown here for the second cycle qualitatively repeats the previous dependence with values of ρ_3/ρ_1 about five times lower. These dependences are obtained at $p_a/p_\infty \approx 860$. The data for high values of the nonratedness show considerably larger values of ρ_3/ρ_1 at the corresponding values of Re_L.

4. Condensation Effects during Expansion of the Gas

The analysis of the process of expansion in the core of the jet, especially in the case of carbon dioxide, is hindered by the indeterminacy in the estimate of the adiabatic index when one attempts to take into account the effect of vibrational relaxation and condensation. Judging from the vibrational relaxation times obtained on shock tubes, the vibrational energy beyond the critical cross section of the nozzle in our case is frozen in, although the supply of energy from the vibrational degrees of freedom is not fully excluded. The adiabatic index must lie in the range of from 1.3 to 1.4. Nucleation (the formation of clusters in the initial stage of condensation) leads to a decrease in the adiabatic index. On the basis of the equilibrium parameters of the expansion the condensation must begin in the core of the jet in all the modes, and in the nozzle in mode 3b. Because of the high expansion rate, however, the condensation process does not proceed in equilibrium and the supersaturation can be very considerable.

For CO_2 jets beyond supersonic nozzles, according to the data of [18], the conditions of nucleation are not altered when the complexes

$$p_0 d^{0.6} = \text{const} \text{ and } p_0 T_0^{(1.25\gamma - 0.5)/(1-\gamma)} = \text{const} \qquad (4.1)$$

are conserved. The authors of [18] determined a certain effective size of a cluster by the value $(N/Z)^*$, which is the lower limit of the sizes of charged clusters crossing a stopping potential barrier and being

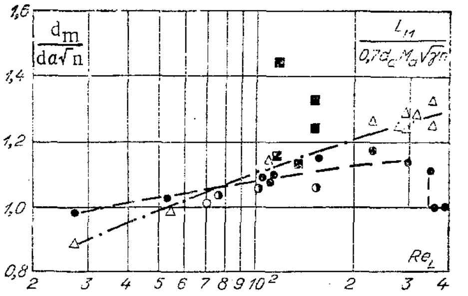

Fig. 9

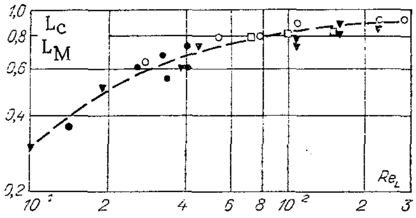

Fig. 10

recorded by a mass spectrometer at the level of half the total ion current. At present this is the most accessible characteristic of nucleation on the basis of experimental data, since it gives at least the order of magnitude of the linear dimensions of the clusters. The values of $(N/Z)^*$ found for our conditions from the experimental data of [18] using the functions (4.1) are presented in Table 1.

With the exception of modes 3b for CO_2 and 5a for N_2 the concentration of nuclei in the core of the jet, according to the estimates of [19], is too low to have an effect on the energy distribution in the stream. Therefore, in the analysis of the experiments of the present work on the effect of the viscosity we took γ = 1.4 for nitrogen and γ=1.3 for CO_2 with a certain arbitrariness in terms of the allowance for the vibrational–rotational and vibrational–translational interactions.

The effect on the structure of the jet of the condensation in the course of expansion was not studied in the present work, although certain modes were especially chosen to display the condensation effects. Photographs of plane cross sections of CO_2 jets in modes 3a and b, characterized by close pressure ratios p_0/p_∞ and almost the same Reynolds numbers Re_L, are shown in Fig. 7. The case *a* is without condensation; in case b the condensation evidently begins in the nozzle. The diameter of the suspended shock wave is markedly larger than in the jet without condensation (case *a*); this may correspond to smaller values of M_a than in case *a*; the X-shaped configuration changes into a Mach configuration and the intensity of the shock waves of the second cycle decreases. The relative values of the density in the core of the jet are almost twice as small in the presence of condensation.

We should note that the intensive condensation in the presence of condensed particles in the stream cannot be taken as the immediate cause of the appearance of the Mach disk in the case under consideration; apparently because of the condensation the gasdynamic parameters of the stream (including the flow in the nozzle) were altered in such a way that the X-shaped configuration proved to be unstable.

5. Self-Similarity of the Density Distribution and of the Characteristic Dimensions of the Jets

Let us choose the self-similar coordinates for the density and the distance along the jet axis by analogy with [1]: $\rho=\rho/(\rho_0)\cdot 0{,}7\ M_a\sqrt{\gamma n}$ and $\bar{x}^1=x/(0.7d_aM_a\sqrt{\gamma n})$. In Fig. 8a, b the density distribution in a nitrogen jet is shown in these coordinates by modes 4a-c and 4h-j for Re_L=93 and 37, respectively. As follows from Fig. 3b, the generalization is fully satisfactory for n > 1000. Since it extends beyond the region of joining of the shock waves, this indicates the similarity of the geometry of the suspended shock waves and of the density distribution in the compressed layer. Generalization is not observed in the region of the X-configuration with lower values of the nonratedness (Fig. 8a; n=190-580). The deviation of the geometry of a nonviscous jet from self-similarity already becomes important at such values of n, as follows from Eq. (1.1).

It is known that in jets where the effect of the mixing zone in the initial section of the jet does not extend to its axis the distance to the Mach disk obeys the dependence [1, 20]

$$L_M=0.7\ d_aM_a\sqrt{\gamma n}. \tag{5.1}$$

As the experiments showed, the transition to an X-shaped configuration occurs abruptly with a decrease in Re_L; when this happens the distance along the axis of the jet from the nozzle cut to the shock waves increases abruptly and then it decreases in accordance with the value of Re_L. This is illustrated by Fig. 9, where the dependence $L_M/0.7M_ad_a\sqrt{\gamma n}=f(Re_L)$ is given for a CO_2 jet in modes 2a-j with n ≈ 860 (blackened circles and dashed average curve). The value of L_M increases by 15-20% upon the change in configuration, and for $\mathrm{Re}_L < 40$ it becomes shorter than the distance to the Mach disk. Equation (5.1) can also be used for an approximate quantitative estimate of the distance to the shock waves along the jet axis in the presence of the X-configuraion. The data in Fig. 9 are characterized by order-of-magnitude errors.

We note that in [5, 6] overstated values of L_M (and of the maximum diameter d_m of the suspended shock waves) were obtained from measurements of a glow-discharge on photographs, and a dependence of L_M on M_a was proposed which was stronger than that based on Eq. (5.1). Evidently, the distribution of the emission in a glow-discharge is ambiguously connected with the density distribution in the jet of a neutral gas, which may be responsible for the anomalous results of the reports cited.

The effect of the number Re_L on the ratio of the geometrical dimensions of the jet follows from the data of Fig. 9. Here the values for modes 2a-j are presented (light triangles and dot-dashed averaged curve). As is seen, geometrical similarity is not preserved; the ratio L_M/d_m increases considerably with a decrease in Re_L, and the core of the jet becomes relatively thinner.

In order to analyze the self-similarity of the geometrical configuration of the jet with $\mathrm{Re}_L = \mathrm{const}$ the modes 4d-g with $\mathrm{Re}_L \approx 66$ were selected from the experimental data for nitrogen and the dimensionless complex $d_m/d_a\sqrt{n}$ was calculated for them from the photometric data. It was found that $d_m/d_a\sqrt{n} = \mathrm{const} = 1.01$ in the range of n=231-1140, while the ratio of the characteristic dimensions is $d_m/d_M = \mathrm{const} = 0.22$. Calculations for an ideal gas [11] give $d_m/d_a\sqrt{n} \approx 1.21$ for the conditions $M_a = 5$, $\gamma = 1.4$, $n = 10^3$, and $\Theta = 10°$; the diameter of the suspended shock wave with $\mathrm{Re}_L \approx 66$ is 20% smaller in comparison with a jet of nonviscous gas.

In contrast to a nitrogen jet, the geometry of a CO_2 jet is nonself-similar with $\mathrm{Re}_L = \mathrm{const}$. The values of $d_m/d_a\sqrt{n}$ for modes 1b, c, h, i, j, and 2h in the range of Reynolds numbers of 107-171 are shown in addition in Fig. 9 by blackened squares. The tendency of the dimaeter of the suspended shock waves to increase with an increase in the nonratedness with close Reynolds numbers is clearly traced. The conditions with respect to condensation are about the same for these modes. Evidently the reason for the absence of self-similarity consists in the properties of the energy exchange of the internal and translational degrees of freedom.

The effect of the temperature factor on the dimensions of the jet are only touched on in the present report. On the example with nitrogen it is shown that in the range of $52 < \mathrm{Re}_L < 104$ with the nonratedness kept approximately constant the relative density distribution along the jet axis does not undergo a significant change upon the heating of the gas up to 950°K. The slight tendency of a shift of the second cycle (by no more than 10% downstream) with the transition from mode 5b to 5d is in agreement with the effect of the viscosity described above (see Fig. 9).

In the practical use of low-density jets the question arises of the dimensions of the core of the jet which is not disturbed by the shock-wave fountains. The data on the length of the core of the jet obtained in the present work and the experimental data available in the literature [6, 21] are summarized in Fig. 10. The point with a 10% deviation in the density from the isentropic value was taken as the boundary of the core of the jet. The connection between the dimension L_c of the core and the number Re_L can be represented by the dependence $L_c/L_M = (1 + 20/\mathrm{Re}_L)^{-1}$.

The authors thank A. V. Ivanov for useful discussions.

LITERATURE CITED

1. V. S. Avduevskii, A. V. Ivanov, I. M. Karpman, V. D. Traskovskii, and M. Ya. Yudelovich, "Flow in a supersonic, viscous, underexpanded jet," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3 (1970).
2. V. S. Avduevskii, A. V. Ivanov, I. M. Karpman, V. D. Traskovskii, and M. Ya. Yudelovich, "Effect of viscosity on the flow in the initial section of a strongly underexpanded jet," Dokl. Akad. Nauk SSSR, 197, No. 1 (1971).
3. C. E. Peters and W. I. Phares, "The structure of plumes from moderately underexpanded supersonic nozzles," AIAA Paper, No. 70, 229 (1970).
4. V. I. Nemchenko, "Study of the closing shock wave in a submerged, supersonic, underexpanded gas jet," Inzh.-Fiz. Zh., 20, No. 5 (1971).
5. S. A. Senkovenko, "Structure of underexpanded superonic CO_2 jets," in: Proceedings of Second Republic Conference on Aerodynamics, Heat Exchange, and Mass Exchange. High-Velocity Aerodynamics Section [in Russian].

6. V. V. Volchkov, A. V. Ivanov, N. I. Kislyakov, A. K. Rebrov, V. A. Sukhnev, and R. G. Sharafutdinov, "Low-density jets beyond a sonic nozzle with large pressure drops," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1973).
7. E. P. Muntz, B. B. Hamel, and B. L. Maguire, "Some characteristics of exhaust plume rarefaction," AIAA J., 8, No. 9 (1970).
8. J. P. Moran, "Similarity in a high-altitude jet," AIAA J., 5, No. 7 (1970).
9. V. N. Gusev and V. V. Mikhailov, "On the similarity of flows with expanding jets,"Uch. Zap. Tsentr. Aéro-Gidrodin, Inst., 1, No. 4 (1970).
10. I. N. Murzinov, "Similarity parameters in the discharge of strongly underexpanded jets into a flooded space," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4 (1971).
11. G. I. Averenkova, É. A. Ashratov, T. G. Volkonskaya, Yu. N. D'yakonov, N. I. Egorova, D. A. Mel'nikov, G. S. Roslyakov, and V. I. Uskov, Supersonic Jets of an Ideal Gas [in Russian], Part 2, Izd. Mosk. Gos. Univ., Moscow (1971).
12. Yu. P. D'yakonov and V. I. Uskov, "Calculation of supersonic jets of an ideal gas by the wall method," Nauchno. Trudy Nauchno.-Issled. Inst. Mekh. Mosk. Gos. Univ., No. 5 (1970).
13. N. I. Kislyakov, A. K. Rebrov, and R. G. Sharafutdinov, "Diffusional processes in the mixing zone of a supersonic low-density jet," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1973).
14. E. P. Munts, "Measurement of rotational temperature, vibrational temperature, and molecule concentration in nonradiating flows of low density nitrogen," UTIAS Rep., No. 71 (1961).
15. V. A. Kosinov, A. A. Bochkarev, V. G. Prikhod'ko, and A. K. Rebrov, "Measurement of the density of CO_2, CO, H_2, and H_2O gases using an electron beam," in: Scientific Proceedings of the Institute of Heat Physics, Siberian Branch of the Academy of Sciences of the USSR [in Russian], Novosibirsk (1972).
16. A. Ferri, Aerodynamics of Supersonic Flows [Russian translation], Inostr. Lit., Moscow (1952).
17. D. A. Mel'nikov, "Reflection of shock waves from the axis of symmetry," Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekh. Mashinostr., No. 3 (1962).
18. O. F. Hagena and W. Obert, "Cluster formation in expanding supersonic jets: effect of pressure, temperature, nozzle size, and test gas," J. Chem. Phys., 56, No. 5 (1972).
19. R. M. Velland, I. M. Deckers, I. D. Scott, and Tiori, "Dimer concentrations in supersonic free jets," Canad. J. Phys., 5, No. 20 (1972).
20. C. H. Lewis and D. I. Carlson, "Normal shock location in underexpanded gas and gas-particle jets" AIAA J., 2, No. 4 (1964).
21. A. K. Rebrov, S. F. Chekmarev, and R. G. Sharafutdinov, "Effect of rarefaction on the structure of a free nitrogen jet," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1971).